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ON DETERMINING ABSOLUTE COORDINATE SYSTEMS WITH THE AID OF PLAN--ETC(U)
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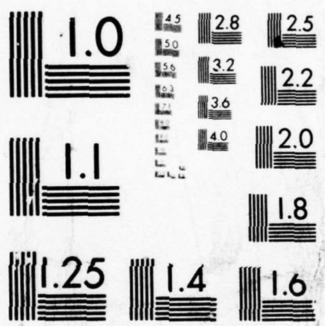

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ON DETERMINING ABSOLUTE COORDINATE SYSTEMS WITH THE AID OF PLANET OBSERVATIONS

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BY F. SCHMEIDLER, MUNICH

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The equations of condition for the improvement of the orbital elements of planets from meridian observations are transformed in such a manner that only the geocentric coordinates ofhthe planets and of the sun appear in the coefficients. For planets, which move in orbits with little excentricity and inclination, the equations can be written in the convenient form

$$\Delta\alpha = -E + b_0 + a_0 (t - t_0) + a_1 \sin\alpha + b_1 \cos\alpha - a_2 \operatorname{tg}\varepsilon \sin\alpha \cos\alpha - b_2 \operatorname{tg}\varepsilon \cos^2\alpha$$
$$\Delta\delta = -D + a_0 (t - t_0) \operatorname{tg}\varepsilon \cos\alpha + a_2 \sin\alpha + (b_2 + b_0 \operatorname{tg}\varepsilon) \cos\alpha \quad a_1 \operatorname{tg}\varepsilon \sin\alpha \cos\alpha + b_1 \operatorname{tg}\varepsilon \cos^2\alpha$$

in this case, the coefficients a_i, b_i are also a function of the quantity $\alpha - \alpha_\odot$, which is a measure for the time period before and after the opposition. The observation material is subdivided into groups in wuch a manner that, in each group, the value of $\alpha - \alpha_\odot$, and thus also the values of a_i, b_i are constant. As a rule, only the systems corrections E and D, which are characteristic for instruments and observers, are of interest; the determination of the orbital elements of the planets can mostly be left to later conversion.

The simplified form of the equations makes possible a clear discussion, as to which of the unknowns, under which conditions, can be reliably separated. A numerical example shows that the assumption of a small excentricity and inclination is also still permissible for Mars, where maximum errors should be expected because of the shape of the orbit and the nearness of the earth. The observation of large planets made with the

Munich meridian instruments in the years 1941 - 1956 are treated with the aid of the described method.

1. GENERAL COMMENTS

The determination of the corrections of the equinox and of the equator point, necessary for the derivation of absolute star loci, is usually carried out with the aid of observations of the sun and occasionally also of the planets Mercury and Venus. However, it is known that these observations, which must be made during the day, are less accurate and more susceptible to systematic errors than observations of fixed stars. For this reason, observations of small and also large outer planets have been proposed as alternatives several times and various astronomical yearbooks publish accurate ephemerides of Ceres, Pallas, Juno and Vesta since 1952.

The use of planet observations for the definition of absolute coordinates is faced with the difficulty that no less than 12 unknowns must be determined. In addition, the calculation of the coefficients of the equations of condition is cumbersome and time-consuming. A fictitious example, which was calculated by Clmence (1) shows how extensive the necessary calculation is. Finally, the disadvantage should also be emphasized that, in the customary method of the simultaneous determination of the orbital elements, both of the planet and of the earth, as well as of the corrections of the equinox and the equator, it is not always possible to recognize at first glance which of the unknowns are difficult to separate from the others.

For this reason, the author considered it desirable to derive a system of formulas, which is easy to handle numerically and which additionally allows a clear overview of the conditions of the separability of the various unknowns. For observations of the sun, these conditions are adequately fulfilled by the known system

$$\left.\begin{aligned} \Delta\alpha_{\odot} &\mp -E + \cos\varepsilon \sec^2\delta_{\odot}\, \Delta L' - \cos\alpha_{\odot}\, \mathrm{tg}\,\delta_{\odot}\, \Delta\varepsilon + 2\sin\alpha_{\odot} \sec\delta_{\odot}\, \Delta h' - 2\cos\varepsilon \cos\alpha_{\odot} \sec\delta_{\odot}\, \Delta k', \\ \Delta\delta_{\odot} &\mp -D + \sin\varepsilon \cos\alpha_{\odot}\, \Delta L' + \sin\alpha_{\odot}\, \Delta\varepsilon + 2\cos\alpha_{\odot} \sin\delta_{\odot}\, \Delta h' - 2\sin\varepsilon \cos^2\alpha_{\odot} \cos\delta_{\odot}\, \Delta k', \end{aligned}\right\} \quad (1)$$

in which E and D signify the sought corrections of equinox and equator.

Hough (2) derived a convenient method for the inner planets Mercury and Venus, requiring only the determination of some principal terms of trigonometric series. Numerow (3) discussed the treatment of observations of outer planets in several papers; however, the formulas developed by him are not less cumbersome and vague than the customary terms of the theory of the orbit improvement. In contrast, a simplification could be obtained primarily when the coefficients of the equations of condition would not be illustrated as functions fo the true anomaly, but of the geocentric coordinates, because the latter can be taken directly from the ephemeride.

In the theoretical portion, this paper has the objective of illustrating the differences between the observed and the calculated loci of the outer planets as linear functions of the unknowns in such a manner that, if possible, only the geocentric right ascension appears as argument. It will be shown that true simplifications can be obtained only by vigorous neglect. On the other hand, it would be senseless to seek an accuracy of more than 10 percent; the differences between observation and calculation, with good ephemerides, are of the order of magnitude of 1", and the average error of the unknown, which are obtained by the resolution of the equations of condition by the method of the least squares, are mostly about ±0"I. Therefore, it appears obvious to strike all terms in the formulas not greater than 10 percent of the principal terms. In all cases, in which a greater accuracy is required or is sought for specific reasons, it is probably unavoidable to carry out the calculations in accordance with the more rigorous formulas.

The neglects which have been implemented will not only provide relatively simple formulas, but it will also become clear which of the unknowns are coupled with which others in a manner which is separable with difficulty. In addition, the simplified terms make it easy to recognize which of the unknown can be determined with certainty only when the observations of the planet cover a sufficiently extensive period of time prior to and after the opposition. Finally, the formalism developed in this paper provides the possibility of obtaining at least as much information about the unknowns from observations, which are made only in the vicinity of the opposition, as will be possible on the basis of the prevailing conditions.

2. INFLUENCE OF THE ERRORS OF THE ORBIT ELEMENTS OF THE PLANET

Because this concerns the use of observations of equatorial coordinates, all orbit elements and definitions are referred to the equator; the elements of the orbit of the planet are thus

- M_0 = average anomaly to the time t_0
- μ = average daily motion
- a = great semiaxis, with μ connected by $\mu^2 a^3 = k^2(I+m)$
- e = $\sin\varphi$ = excentricity
- N = ascending node on the equator
- w = angle between node and perihelion
- J = inclination of the orbit with respect to the equator.

In the conventional manner, the heliocentric distance should finally be designated as r, the true anomaly as v and the geocentric distance as ϱ; in addition, the abbreviating quantity $u = v + w$ shall be defined in the same manner as in the ecliptic calculation. In accordance with Bauschinger (4), the equations for the determination of the errors of the orbit elements are:

$$
\left.
\begin{aligned}
\cos\delta\,\Delta\alpha = {} & \frac{a}{\varrho}\left(\sin b\cos(B+u) + e\sin b\cos(B+w)\right)\sec\varphi\,(\Delta M_0 + (t-t_0)\,\Delta\mu) \\
& - \frac{2a\sqrt{a}}{3k}\frac{r}{\varrho}\sin b\sin(B+u)\,\Delta\mu \\
& + \frac{a}{\varrho}\left(\sin E\sin b\cos(B+u) - \cos\varphi\sin b\sin(B+w)\right)\Delta\varphi \\
& + \frac{r}{\varrho}\sin b\cos(B+u)\,\Delta s + \frac{r}{\varrho}\cos b\sin u\,\Delta J - \frac{r}{\varrho}\cos b\cos u\sin J\,\Delta N\,. \\
\Delta\delta = {} & \frac{a}{\varrho}\left(\sin c\cos(C+u) + e\sin c\cos(C+w)\right)\sec\varphi\,(\Delta M_0 + (t-t_0)\,\Delta\mu) \\
& - \frac{2a\sqrt{a}}{3k}\frac{r}{\varrho}\sin c\sin(C+u)\,\Delta\mu \\
& + \frac{a}{\varrho}\left(\sin E\sin c\cos(C+u) - \cos\varphi\sin c\sin(C+w)\right)\Delta\varphi \\
& + \frac{r}{\varrho}\sin c\cos(C+u)\,\Delta s + \frac{r}{\varrho}\cos c\sin u\,\Delta J - \frac{r}{\varrho}\cos c\cos u\sin J\,\Delta N\,.
\end{aligned}
\right\} \quad (2)
$$

In this case, instead of the quantities Δp, Δq, used by Bauschinger, the direct corrections ΔJ, ΔN of the elements are introduced; the following relationships apply

$$
\left.
\begin{aligned}
\Delta p &= \sin J\sin w\,\Delta N + \cos w\,\Delta J\,, \\
\Delta q &= \sin J\cos w\,\Delta N - \sin w\,\Delta J\,, \\
\Delta s &= \cos J\,\Delta N + \Delta w\,.
\end{aligned}
\right\} \quad (3)
$$

The auxiliary quantities b, B are defined by the system

$$
\left.
\begin{aligned}
\sin b\sin B &= -\sin(\alpha - N)\,, \\
\sin b\cos B &= \cos J\cos(\alpha - N)\,, \\
\cos b &= -\sin J\cos(\alpha - N)
\end{aligned}
\right\} \quad (4)
$$

while, for c, C, the relationships

$$
\left.
\begin{aligned}
\sin c\sin C &= -\sin\delta\cos(\alpha - N)\,, \\
\sin c\cos C &= \sin J\cos\delta - \cos J\sin\delta\sin(\alpha - N)\,, \\
\cos c &= \cos J\cos\delta + \sin J\sin\delta\sin(\alpha - N)
\end{aligned}
\right\} \quad (5)
$$

apply as definition.

In order to avoid the known uncertainty for small excentricities and inclinations, the new unknowns are better used

$$
\left.
\begin{aligned}
\Delta L_0 &= \sec^3\varphi\,\Delta M_0 + \Delta s\,, \\
\Delta v &= e\,\Delta s\,, \\
\Delta K &= \sin J\,\Delta N\,.
\end{aligned}
\right\} \quad (6)
$$

In (2), the semimajor axis a in the coefficients of $\Delta M_0 + (t-t_0)$ should additionally be replaced by the expression $r \sec^2\varphi\,(1+e\cos v)$ and, in the coefficients of $\Delta\varphi$ the factor a sin E by the expression $r \sec\varphi \sin v$. If u - v is finally also written in (2) instead of w, the following form of the equations of condition is obtained:

$$\left.\begin{aligned}
\varrho\cos\delta\,\Delta\alpha &= r\sin b\,(P\sin(B+u) + Q\cos(B+u)) + r\cos b\,(\sin u\,\Delta J - \cos u\,\Delta K)\,,\\
\varrho\,\Delta\delta &= r\sin c\,(P\sin(C+u) + Q\cos(C+u)) + r\cos c\,(\sin u\,\Delta J - \cos u\,\Delta K)\,,\\
P &= -\frac{2a\sqrt{a}}{3k}\,\Delta\mu + (1+e\cos v)\,(e\sin v\,\Delta L - \sin v\,\Delta v) - \frac{a}{r}\cos\varphi\cos v\,\Delta\varphi\,,\\
Q &= \Delta L + (2+e\cos v)\,(e\cos v\,\Delta L - \cos v\,\Delta v) + \left(\sec\varphi + \frac{a}{r}\cos\varphi\right)\sin v\,\Delta\varphi\,,\\
\Delta L &= \Delta L_0 + (t-t_0)\sec^3\varphi\,\Delta\mu\,.
\end{aligned}\right\}\quad (7)$$

In equations (7), r, u and v must be expressed by the geocentric coordinates of the planet and of the sun. It is known that

$$\left.\begin{aligned}
x &= \varrho\cos\alpha\cos\delta - R\cos\alpha_\odot\cos\delta_\odot = r\,(\cos N\cos u - \sin N\sin u\cos J)\,,\\
y &= \varrho\sin\alpha\cos\delta - R\sin\alpha_\odot\cos\delta_\odot = r\,(\sin N\cos u + \cos N\sin u\cos J)\,,\\
z &= \varrho\sin\delta \qquad\quad\;\; - R\sin\delta_\odot \qquad\quad\;\; = r\sin u\sin J\,,
\end{aligned}\right\}\quad (8)$$

where R, $\alpha_\odot$, $\delta_\odot$ are the geocentric coordinates of the sun. If the two auxiliary quantities

$$\xi = \frac{R\cos\delta_\odot}{\varrho\cos\delta}\cos(\alpha-\alpha_\odot)\,,\qquad \eta = \frac{R\cos\delta_\odot}{\varrho\cos\delta}\sin(\alpha-\alpha_\odot) \qquad (9)$$

are introduced, these relations are obtained from (8)

$$\left.\begin{aligned}
r\sin u &= \varrho\,(1-\xi)\cos\delta\sin(\alpha-N)\sec J + \varrho\,\eta\cos\delta\cos(\alpha-N)\sec J\,,\\
r\cos u &= \varrho\,(1-\xi)\cos\delta\cos(\alpha-N) - \varrho\,\eta\cos\delta\sin(\alpha-N)\,.
\end{aligned}\right\}\quad (10)$$

Formulas (10) make possible the calculation of r, u and v = u - w from the geocentric coordinates of the planet, which can be taken from the ephemeride. The two parameters ξ and η are essentially a function of $\alpha - \alpha_\odot$ and are a measure of the time interval of the observations from the opposition.

Because most planets move reasonably close to the ecliptic, it is useful to express sin δ as a function of α. Using the

second equation (10), the following relationship is found from the third equation (8)

$$\varrho \sin \delta = \varrho\,(1 - \xi)\,\mathrm{tg}\, J \cos \delta \sin (\alpha - N) + \varrho\,\eta\,\mathrm{tg}\, J \cos \delta \cos (\alpha - N) + R \sin \delta_{\odot}.$$

Because the known relationship $\tan \delta_{\odot} = \tan \varepsilon \sin \alpha_{\odot}$ exists, the following is obtained

$$R \sin \delta_{\odot} = R\,\mathrm{tg}\,\varepsilon \cos \delta_{\odot} \sin \alpha_{\odot} = \varrho\,\mathrm{tg}\,\varepsilon \cos \delta\,(\xi \sin \alpha - \eta \cos \alpha).$$

If, herein, α is replaced by N + (α - N), the following expression is obtained for sin δ

$$\left.\begin{aligned} &\sin \delta = \mathrm{tg}\, J \cos \delta \sin (\alpha - N) - \gamma \\ \text{with}\quad &\gamma = (\xi\,(\mathrm{tg}\, J - \cos N\,\mathrm{tg}\,\varepsilon) - \eta \sin N\,\mathrm{tg}\,\varepsilon) \cos \delta \sin (\alpha - N) - (\eta\,(\mathrm{tg}\, J - \cos N\,\mathrm{tg}\,\varepsilon) \\ &\quad + \xi \sin N\,\mathrm{tg}\,\varepsilon) \cos \delta \cos (\alpha - N). \end{aligned}\right\} \quad (11)$$

For planets, which move exactly in the ecliptic, J = ε and N = 0, and consequently also = 0. For planets, of the which the orbit inclination with respect to the ecliptic is small, γ can be considered as a small quantity and can be neglected completely when the requirements for accuracy are small.

The advantage of relationship (11) for sin δ consists of the fact it allows the expression of the quantities c, C as functions of b, B and do therefore not have to be calculated separately. If expression (11) for sin δ is inserted in the two first equations (5), the following is found

$$\sin c \sin C = \mathrm{tg}\, J \cos \delta \cos (\alpha - N) \sin b \sin B + \gamma \sec J \sin b \cos B,$$
$$\sin c \cos C = \mathrm{tg}\, J \cos \delta \cos (\alpha - N) \sin b \cos B - \gamma \cos J \sin b \sin B.$$

In the terms, which are multiplied with the small quantity γ, sec J = cos J = 1 can be inserted, and it follows

$$\left.\begin{aligned} r \sin c \sin (C - u) &= r\,\mathrm{tg}\, J \cos \delta \cos (\alpha - N) \sin b \sin (B + u) + \gamma\, r \sin b \cos (B + u), \\ r \sin c \cos (C + u) &= r\,\mathrm{tg}\, J \cos \delta \cos (\alpha - N) \sin b \cos (B + u) - \gamma\, r \sin b \sin (B + u). \end{aligned}\right\} \quad (12)$$

In addition, an expression for cos c can be found, when (11) is inserted in the third equation (5)

$$\cos c = \cos J \cos \delta + \sin J\,\mathrm{tg}\, J \cos \delta \sin^2 (\alpha - N) - \gamma \sin J \sin (\alpha - N).$$

By neglecting the square of γ, the following can be written

$$\cos c = \cos J \cos\delta\,(1 + \mathrm{tg}^2\delta) + \gamma \sin J \sin(\alpha - N) = \cos J \sec\delta + \gamma \sin J \sin(\alpha - N) . \quad (13)$$

All quantities, which occur in the equations of condition (7), can be calculated with the aid of formulas (4), (9), (10), (12) and (13). The derived formulas are completely formal, except that, in the terms multiplied with the small quantity γ, sec J = cos J = 1 is inserted and γ^2 is neglected. In most cases, γ = 0 can also be assumed.

3. INFLUENCE OF THE ERRORS OF THE ORBIT ELEMENTS OF THE EARTH

As a rule, only four earth orbit elements are considered as requiring improvement. The following unknowns are used

$\Delta L'$ = correction of the mean longitude of the sun
$\Delta\epsilon$ = correction of the skew of the ecliptic
$\Delta h'$ = correction of the quantity $h' = e' \cos\pi'$
$\Delta k'$ = correction of the quantity $k' = e' \sin\pi'$.

Newcomb (5) derived the formulas, which represent the influence of the corrections of the earth orbit elements on the planet loci. However, an independent treatment of the problem appears to be more suitable for the purposes of the present work. As a starting point, equations (8) can be used in the form

$$\left.\begin{aligned} \varrho\cos\alpha\cos\delta &= x + x' , \\ \varrho\sin\alpha\cos\delta &= y + y' , \\ \varrho\sin\delta &= z + z' \end{aligned}\right\} \quad (14)$$

in which x', y', z' are the geocentril rectangular equatorial coordinates of the sun. They are given by the expressions

$$\left.\begin{aligned} x' &= R\cos\alpha_\odot\cos\delta_\odot = R\cos\lambda_\odot , \\ y' &= R\sin\alpha_\odot\cos\delta_\odot = R\cos\varepsilon\sin\lambda_\odot , \\ z' &= R\sin\delta_\odot \qquad\quad\;\; = R\sin\varepsilon\sin\lambda_\odot . \end{aligned}\right\} \quad (15)$$

The geocentric coordinates x, y, z of the planet must now be considered as constant. Through differentiation and elimination

of $\Delta\varrho$, the following differential equations are obtained in a known manner from equations (14)

$$\varrho \cos\delta\, \Delta\alpha = \cos\alpha\, \Delta y' - \sin\alpha\, \Delta x',$$
$$\varrho\, \Delta\delta = \cos\delta\, \Delta z' - \cos\alpha \sin\delta\, \Delta x' - \sin\alpha \sin\delta\, \Delta y'.$$

However, since the relation z' = y' tanε applies in accordance with equations (15), Δz' can be replaced by $\tan\varepsilon\, \Delta y' + y' \sec^2\varepsilon\, \Delta\varepsilon$ and the following is found

$$\left.\begin{aligned} \varrho \cos\delta\, \Delta\alpha &= \cos\alpha\, \Delta y' - \sin\alpha\, \Delta x', \\ \varrho\, \Delta\delta &= y' \sec^2\varepsilon \cos\delta\, \Delta\varepsilon - \cos\alpha \sin\delta\, \Delta x' + (\operatorname{tg}\varepsilon \cos\delta - \sin\alpha \sin\delta)\, \Delta y'. \end{aligned}\right\} \quad (16)$$

With the aid of equations (15), Δx' and Δy' can be expressed as functions of ΔR, $\Delta\lambda_{\odot}$ and $\Delta\varepsilon$, and

$$\left.\begin{aligned} \Delta x' &= x' \frac{\Delta R}{R} - y' \sec\varepsilon\, \Delta\lambda_{\odot}, \\ \Delta y' &= y' \frac{\Delta R}{R} + x' \cos\varepsilon\, \Delta\lambda_{\odot} - y' \operatorname{tg}\varepsilon\, \Delta\varepsilon. \end{aligned}\right\} \quad (17)$$

is obtained.

The interrelation between the coordinates R, $\lambda_{\odot}$ and the orbit elements of the earth is given by the known formulas

$$R = a'(1 - h' \cos\lambda_{\odot} - k' \sin\lambda_{\odot}); \qquad \lambda_{\odot} = L' + 2\,h' \sin\lambda_{\odot} - 2\,k' \cos\lambda_{\odot}$$

in which the square of the excentricity of the earth orbit is neglected. Through differentiation

$$\left.\begin{aligned} \Delta R &= -x'\, \Delta h' - y' \sec\varepsilon\, \Delta k', \\ \Delta\lambda_{\odot} &= \Delta L' + \frac{2\,y'}{R} \sec\varepsilon\, \Delta h' - \frac{2\,x'}{R}\, \Delta k'. \end{aligned}\right\} \quad (18)$$

is obtained.

These expressions must be inserted in (17) and the result is

$$\left.\begin{aligned} \Delta x' &= -y' \sec\varepsilon\, \Delta L' - \frac{x'^2 + 2\,y'^2 \sec^2\varepsilon}{R}\, \Delta h' + \frac{x'\,y' \sec\varepsilon}{R}\, \Delta k', \\ \Delta y' &= x' \cos\varepsilon\, \Delta L' + \frac{x'\,y'}{R}\, \Delta h' - \frac{y'^2 \sec\varepsilon + 2\,x'^2 \cos\varepsilon}{R}\, \Delta k' - y' \operatorname{tg}\varepsilon\, \Delta\varepsilon. \end{aligned}\right\} \quad (19)$$

These expressions must be inserted in (16) in order to obtain the illustration of $\Delta\alpha$ and $\Delta\delta$ as functions of the improvements of the earth orbit elements. The result of the substitution

shall not be calculated in greater detail at this point, because the formulas become very cumbersome, but they do take on a simpler form with the aid of the neglect in the later part of this paper. However, it should be noted that the coefficients in (19) are fundamentally a function of the rectangular coordinates of the sun, which can be taken from yearbooks.

4. INTRODUCTION OF SIMPLIFICATIONS

The formulas developed under 2. and 3. provide the coefficients of the equations of condition as functions of the geocentric coordinates of the planet and of the sun; the calculation of the locus of the planet in its orbit can be avoided with their aid. Yet, the required calculations should not be significantly less than with the use of the conventional system (2). However, a considerable simplification can be obtained if the excentricities and the square of the inclination are neglected. Because most planets move in the vicinity of the ecliptic, J is approximately ε and the assumption $J^2 = 0$ is just tolerable with the requirement for an accuracy of about 10 percent, which was discussed in the introduction.

When J^2 is neglected, sec J = cos J = 1 and the following is found from equations (4) and (10)

$$\left.\begin{aligned} r \sin b \sin (B + u) &= \varrho\, \eta \cos \delta\,, \\ r \sin b \cos (B + u) &= \varrho\, (1 - \xi) \cos \delta\,. \end{aligned}\right\} \qquad (20)$$

Because of $J \approx \varepsilon$, the difference $J - \varepsilon$ is of the order of magnitude J^2 and can be neglected; for the same reason, $N \approx 0$, and the product $\sin N \tan \varepsilon$ can be neglected as a quantity of the second order. Finally, under the assumed prerequisites, the declinations of the sun and of the planet are greater than ε by at most insignificant amounts and, therefore, $\cos \delta = \cos \delta_{\odot} = 1$ can be used. Under consideration of (20), the following relations are then obtained from (11) and (12)

$$\left.\begin{aligned} \sin\delta &= \operatorname{tg}\varepsilon\sin\alpha\,,\\ r\sin c\sin(C+u) &= r\operatorname{tg}\varepsilon\cos\alpha\sin b\sin(B+u) = \varrho\,\eta\operatorname{tg}\varepsilon\cos\alpha\,,\\ r\sin c\cos(C+u) &= r\operatorname{tg}\varepsilon\cos\alpha\sin b\cos(B+u) = \varrho\,(1-\xi)\operatorname{tg}\varepsilon\cos\alpha\,. \end{aligned}\right\} \quad (21)$$

With the aid of the same neglects, the expressions

$$\left.\begin{aligned} r\cos b\sin u &= -\varrho\operatorname{tg}\varepsilon\cos\alpha\,(p_1\sin\alpha + q_1\cos\alpha)\,,\\ r\cos b\cos u &= -\varrho\operatorname{tg}\varepsilon\cos\alpha\,(p_1\cos\alpha - q_1\sin\alpha)\,,\\ r\cos c\sin u &= \varrho\,(p_1\sin\alpha + q_1\cos\alpha)\,,\\ r\cos c\cos u &= \varrho\,(p_1\cos\alpha - q_1\sin\alpha) \end{aligned}\right\} \quad (22)$$

can be derived from (4), (10) and (13), in which the auxiliary quantities have the significance

$$\left.\begin{aligned} p_1 &= (1-\xi)\cos N + \eta\sin N\,,\\ q_1 &= -(1-\xi)\sin N + \eta\cos N \end{aligned}\right\} \quad (23)$$

With the assumptions r = a, sec J = cos δ = 1 and under consideration of v = u - w, the following expressions

$$\sin v = \sigma\sin\alpha - \tau\cos\alpha\,,$$
$$\cos v = \tau\sin\alpha + \sigma\cos\alpha\,,$$

are obtained from equations (10) for the true anomaly v, which occurs in the equations of condition (7), in which the auxiliary quantities σ and τ are defined by

$$\left.\begin{aligned} \sigma &= \frac{\varrho}{a}\left((1-\xi)\cos(w+N) + \eta\sin(w+N)\right),\\ \tau &= \frac{\varrho}{a}\left((1-\xi)\sin(w+N) - \eta\cos(w+N)\right). \end{aligned}\right\} \quad (24)$$

If expressions (20), (21), (22) and (24), which are obtained by simplifications, are inserted in the equations of condition (7), then the following illustration for $\Delta\alpha$ and $\Delta\delta$ results

$$\left.\begin{aligned} \Delta\alpha &= A_0(t-t_0) + B_0 + A_1\sin\alpha + B_1\cos\alpha - A_2\operatorname{tg}\varepsilon\sin\alpha\cos\alpha - B_2\operatorname{tg}\varepsilon\cos^2\alpha\,,\\ \Delta\delta &= (A_0(t-t_0) + B_0 + A_1\sin\alpha + B_1\cos\alpha)\operatorname{tg}\varepsilon\cos\alpha + A_2\sin\alpha + B_2\cos\alpha\,. \end{aligned}\right\} \quad (25)$$

In this case, the coefficients A_i, B_i are related to the sought corrections of the orbit elements by the following expressions

$$\left.\begin{array}{l|l} A_0 = (1-\xi)\sec^3\varphi\,\Delta\mu, & B_0 = (1-\xi)\,\Delta L_0 - \dfrac{2a\sqrt{a}}{3k}\eta\,\Delta\mu, \\ A_1 = -\eta(\sigma\,\Delta\nu + \tau\,\Delta\varphi) - 2(1-\xi)(\tau\,\Delta\nu - \sigma\,\Delta\varphi), & B_1 = \eta(\tau\,\Delta\nu - \sigma\,\Delta\varphi) - 2(1-\xi)(\sigma\,\Delta\nu + \tau\,\Delta\varphi), \\ A_2 = p_1\,\Delta J + q_1\,\Delta K, & B_2 = q_1\,\Delta J - p_1\,\Delta K. \end{array}\right\} \quad (26)$$

All auxiliary quantities are defined by equations (9), (23) and (24).

Equations (16) and (19), which describe the influence of the errors of the orbit elements of the earth, can be simplified by the same neglects. Under the assumption of sec ε = cos ε = 1 and using the expression (21) for sin δ,

$$\left.\begin{array}{l} \varrho\,\Delta\alpha = \cos\alpha\,\Delta y' - \sin\alpha\,\Delta x', \\ \varrho\,\Delta\delta = y'\,\Delta\varepsilon - \operatorname{tg}\varepsilon\cos\alpha\,(\sin\alpha\,\Delta x' - \cos\alpha\,\Delta y') = y'\,\Delta\varepsilon + \varrho\operatorname{tg}\varepsilon\cos\alpha\,\Delta\alpha. \end{array}\right\} \quad (27)$$

is obtained from (16). Expressions

$$x' = \varrho\cos\delta\,(\xi\cos\alpha + \eta\sin\alpha), \qquad y' = \varrho\cos\delta\,(\xi\sin\alpha - \eta\cos\alpha). \quad (28)$$

are found from (9) and (15) for the rectangular coordinates of the sun x', y'.

If now again cos ε = cos δ = 1 and $\tan^2\varepsilon = 0$, the following illustration of the deviations caused by the errors of the earth orbit elements, which should now be designated as $\Delta\alpha'$ and $\Delta\delta'$, results from equations (19), (27) and (28):

$$\left.\begin{array}{l} \Delta\alpha' = \xi\,\Delta L' + \dfrac{\eta x' + 2\xi y'}{R}\Delta h' + \dfrac{\eta y' - 2\xi x'}{R}\Delta k' - (\xi\sin\alpha - \eta\cos\alpha)\operatorname{tg}\varepsilon\cos\alpha\,\Delta\varepsilon, \\ \Delta\delta' = \left(\xi\,\Delta L' + \dfrac{\eta x' + 2\xi y'}{R}\Delta h' + \dfrac{\eta y' - 2\xi x'}{R}\Delta k'\right)\operatorname{tg}\varepsilon\cos\alpha + (\xi\sin\alpha - \eta\cos\alpha)\,\Delta\varepsilon. \end{array}\right\} \quad (29)$$

Using the relations (28), the coefficients of these equations can be expressed as functions of α, as well as of ξ and η

$$\left.\begin{array}{l} \dfrac{\eta x' + 2\xi y'}{R} = \dfrac{\varrho}{R}(2\xi^2 + \eta^2)\sin\alpha - \dfrac{\varrho}{R}\xi\eta\cos\alpha; \\ \dfrac{\eta y' - 2\xi x'}{R} = -\dfrac{\varrho}{R}(2\xi^2 + \eta^2)\cos\alpha - \dfrac{\varrho}{R}\xi\eta\sin\alpha. \end{array}\right\} \quad (30)$$

It can be seen from equations (29) and (30) that the deviations $\Delta\alpha'$ and $\Delta\delta'$, which are a consequence of the errors of the orbit elements of the earth, can be illustrated in the same general form (25) as the quantities $\Delta\alpha$ and $\Delta\delta$, which are based on the

errors of the orbit elements of the planet. If, additionally, the corrections E and D for equinox and equator are still added, the result is the general illustration for the difference between the observed and the calculated right ascensions and declinations of a planet

$$\left.\begin{aligned}\Delta\alpha &= -E + b_0 + a_0(t-t_0) + a_1\sin\alpha + b_1\cos\alpha - a_2\,\mathrm{tg}\,\varepsilon\sin\alpha\cos\alpha - b_2\,\mathrm{tg}\,\varepsilon\cos\alpha^2,\\ \Delta\delta &= -D + a_0(t-t_0)\,\mathrm{tg}\,\varepsilon\cos\alpha + a_2\sin\alpha + (b_2 + b_0\,\mathrm{tg}\,\varepsilon)\cos\alpha + a_1\,\mathrm{tg}\,\varepsilon\sin\alpha\cos\alpha + b_1\,\mathrm{tg}\,\varepsilon\cos^2\alpha.\end{aligned}\right\}\quad(31)$$

The significance of the coefficients a_i, b_i follows from equations (26), (29) and (30) through the addition of the quantities derived from the errors of the orbits of the planet, respectively of the earth

$$\left.\begin{aligned}a_0 &= (1-\xi)\sec^3\varphi\,\Delta\mu,\\ b_0 &= (1-\xi)\,\Delta L_0 - \frac{2a\sqrt{a}}{3k}\eta\,\Delta\mu + \xi\,\Delta L',\\ a_1 &= -\eta(\sigma\,\Delta v + \tau\,\Delta q) - 2(1-\xi)(\tau\,\Delta v - \sigma\,\Delta q) + \frac{\varrho}{R}(2\xi^2+\eta^2)\,\Delta h' - \frac{\varrho}{R}\xi\eta\,\Delta k',\\ b_1 &= \eta(\tau\,\Delta v - \sigma\,\Delta q) - 2(1-\xi)(\sigma\,\Delta v + \tau\,\Delta q) - \frac{\varrho}{R}\xi\eta\,\Delta h' - \frac{\varrho}{R}(2\xi^2+\eta^2)\,\Delta k',\\ a_2 &= p_1\,\Delta J + q_1\,\Delta K + \xi\,\Delta\varepsilon,\\ b_2 &= q_1\,\Delta J - p_1\,\Delta K - \eta\,\Delta\varepsilon.\end{aligned}\right\}\quad(32)$$

If it is additionally desired to account for a correction Δp of the precision constant and a time dependance of $\Delta L'$ in the form $\Delta L' = \Delta_0 + \Delta_1(t - t_0)$, then the expressions $\Delta p + \xi\Delta_1$ must be added at the right in the first equation (32) and $\Delta L'$ must be replaced by Δ_0 in the second.

The advantage of equations (31) consists of the fact that the coefficients a_i, b_i are constant when $\alpha - \alpha_\odot$ has the same value for all observations. Actually, with the neglects which are introduced, the quantities ϱ, ξ and η are only a function of the semimajor axes of the two orbits and of $\alpha - \alpha_\odot$. With the assumptions $\cos J = \cos\delta = \cos\delta_\odot = 1$ and $e = e' = 0$, consequently $r = a$, the relations

$$\left.\begin{aligned}&\varrho^2 - 2R\varrho\cos(\alpha-\alpha_\odot) + R^2 = a^2,\\ &\xi = \frac{R}{\varrho}\cos(\alpha-\alpha_\odot), \qquad \eta = \frac{R}{\varrho}\sin(\alpha-\alpha_\odot).\end{aligned}\right\}\quad(33)$$

are obtained from equations (9) and (10).

Therefore, for all observations, for which the value $\alpha - \alpha_{\odot}$ is equal or falls into a sufficiently small interval, the coefficients a_i, b_i must be determined through the solution of the system (31). The unknowns themselves then follow from (32). In this manner, the determination of the 12 unknowns is subdivided into several small steps.

5. THE DISCUSSION OF THE FORMULAS

Because the meridian observer is mostly only interested in the systems improvements E and D, the separability of these two quantities shall be discussed first. The determination of D is simplest; it is equal to the constant term in the equation for $\Delta\delta$; uncertainties cannot arise when the material of the observations is good and quite well distributed. A division of the material in accordance with the value $\alpha - \alpha_{\odot}$ is not even absolutely necessary; fitting all observations in accordance with the second equation (31), values are obtained for the quantities a_i, b_i, which represent an approximate average. Again assuming a reasonably uniform distribution of the entire material of the observations, the numerical value of D will be adulterated by insignificant amounts at most.

Contrary to the occasionally expressed opinion, it must be emphasized that the equator correction is not simply equal to the arithmetic mean of all $\Delta\delta$. Because of the factor $\cos^2\alpha$, the last terms in (31) always have the same sign and definitely contribute to the arithmetic mean of all observed values.

The determination of the equinox correction E, which occurs in (31) as the fixed relationship $-E+b_0$, presents somewhat greater difficulties in principle. The quantity b_0 must therefore be determined from the declinations, in which it occurs multiplied by $\tan\varepsilon$, however. For this reason, E, in comparison with the other unknowns, is encumbered with an uncertainty, which is increased by the factor $\cot \varepsilon$. These subjects are not new, but they can be seen particularly clearly with the

simple formulas (31).

In addition, the determination of E poses high requirements on the quality of the observed right ascensions, from which the reliable value of b_2 must be determined; the term with $b_0 \tan\varepsilon$ can only then be separated from b_2 from the declinations. Because b_2 is associated with the mostly small factor $\tan\varepsilon \cos^2\alpha$ in the right ascensions, an additional uncertainty prevails in this manner.

Although the material on the observed right ascensions migght not fulfill the conditions for a reliable determination of b_2, but has a sufficiently good distribution in the variable $\alpha - \alpha_\odot$, the dependence of the coefficients a_i, b_i from this second variable can be utilized in order to obtain a reliable value of E. In this case, the three quantities a_2, $b_3 = b_2 + b_0 \tan\varepsilon$ and $E_0 = -E + b_0$ can be determined with the full reliability, which is associated with the observations. From (32), the relationships

$$\left.\begin{aligned} p_1 \Delta J + q_1 \Delta K + \xi \Delta\varepsilon \qquad\qquad &= a_2, \\ q_1 \Delta J - p_1 \Delta K - \eta \Delta\varepsilon + E \operatorname{tg}\varepsilon &= b_3 - E_0 \operatorname{tg}\varepsilon \end{aligned}\right\} \tag{34}$$

can be derived and the four unknowns ΔJ, ΔK, $\Delta\varepsilon$ and E can be determined when the material allows a subdivision into at least two considerably different values of $\alpha - \alpha_\odot$. This possibility was largely used in the evaluation of the Munich planetary observations, which are further reported below.

The determination of the equinox correction E is configured still more advantageous, when the errors of the orbit elements of the earth are either neglected or can otherwise be assumed as known. In accordance with the second equation (32), the simple relationship for the quantity $-E+b_0$, which is determinable from the right ascensions alone

$$-E + b_0 = E_0 = -E + (1 - \xi)\,\Delta L_0 - \frac{2a\sqrt{a}}{3k}\,\eta\,\Delta\mu, \tag{35}$$

from which E can be determined, if the material of the

observations is distributed over at least two significantly different values of $\alpha - \alpha_{\odot}$ and $\Delta\mu$ is taken from the first equation (32).

The corrections of the orbit elements of the earth and of the planet can be found from (32). Again, as can be seen immediately, the numerical values a_i, b_i are required for at least two significantly different values $\alpha - \alpha_{\odot}$; the occupation of as wide a range in this variable is of course desirable. If the corrections of the earth orbit elements can be neglected, the problem becomes correspondingly simpler.

The simplification, which can be obtained through the neglect of the corrections of the earth orbit, has considerable significance, primarily for the outer planets, which are at a considerable distance. It is obvious that the observations of these planets can provide little information about the orbit elements of the earth; analytically, this is expressed by the fact that, in (32), all corrections of the earth orbit are multiplied with the factors ξ or η, which, corresponding to expressions (33), become very small for very distant planets.

Finally, it is possible to continued in so far as the systematic errors of the instrument can be considered as periodic functions of the right ascensions, thus the errors $\Delta\alpha_{\alpha}$ and $\Delta\delta_{\alpha}$ can be introduced in the form of terms which are proportional to $\sin\alpha$ and $\cos\alpha$. The amplitude of these terms would have be added in the system (32) in the appropriate equations. It can be seen that these terms can be reliably separated from the other unknown only when the observations uniformly cover a sufficiently broad range of the variables $\alpha - \alpha_{\odot}$; this is equal to the requirement that the observations must extend over a sufficiently long time period prior to and after the opposition.

6. VERIFICATION OF THE ADMISSIBILITY OF THE SIMPLIFICATIONS

It was tested on the basis of a numerical example whether the neglect of the excentricities and the squares of the inclinations is permissible. As the specimen calculation, the observations of Mars, made in the years 1941 to 1956 with the Munich meridian instruments, were selected, because the greatest errors are suspected in the case of this planet because of its excentricity and nearness to the earth. All declinations are published in the catalog of the author (6) on azimuth circle observations; however, the values $\Delta\delta$ which are given there are already corrected because of the systems corrections, which are here investigated and are therefore different from the values given in this paper. Up to the beginning of 1950, the right ascensions are published by Labitzke (7), (8); in addition, the author owes thanks to Mr. Labitzke for the premature release of unpublished observations of the years 1950-1952.

Table 1 shows the normal loci, which are formed from the material. Three groups were formed, one each for observations with small (A), medium (B) and large (C) value of $\alpha - \alpha_{\odot}$. In the column "location", which occurs only in the declinations, M = Munich and C = Canberra because, in 1954/55, observations were made in Canberra, Australia, using the Munich azimuth circle. All other designations in Table 1 do not require an explanation.

In the distribution of the observations into groups, no fixed ranges of $\alpha - \alpha_{\odot}$ were maintained, but individual observations were absorbed in another group for the purpose of a more advantageous distribution. It proved to be impossible to form more than three groups, which entailed considerable advantages.

The improvements of the orbit elements of the earth can be ignored for the questions which here alone are discussed, whether the leglects which have been carried out are permissible. The

Table 1. Normal Loci of Mars

1 Zeit	2 Ort	n	Gruppe 3	α	$\alpha - \alpha_{\odot}$	$\Delta\alpha$	$\Delta\delta$
				h m	h m	s	″
1944.1	M	3	A	4 20	7 29	−1.50	
44.2	M	4	A	4 54	6 30	−0.98	
48.3	M	2	C	9 28	9 18	−2.32	
48.3	M	2	B	9 26	8 55	−2.85	
48.4	M	3	A	9 37	7 42	−3.25	
52.4	M	4	C	14 8	10 20	−0.90	
52.5	M	2	A	13 58	7 52	+0.15	
1941.9	M	2	B	0 44	9 27		−0.40
41.9	M	3	A	0 51	8 25		+0.23
44.0	M	1	B	4 10	9 3		+0.50
44.1	M	2	A	4 20	7 31		0.00
44.2	M	2	A	4 56	6 26		+0.50
48.2	M	1	C	9 28	9 24		0.00
48.3	M	1	B	9 26	8 50		+0.40
48.3	M	2	A	9 34	7 50		+0.95
50.3	M	2	B	11 34	9 2		+0.65
50.3	M	2	B	11 33	8 42		+0.70
50.4	M	3	A	11 34	8 22		+0.40
52.4	M	2	C	14 4	9 55		+0.65
52.4	M	2	B	13 50	0 1		+0.50
52.5	M	2	A	13 55	8 32		−0.40
54.4	C	2	C	18 33	13 46		+0.30
54.5	C	2	C	18 11	11 58		−0.05
54.5	C	2	B	17 50	10 10		+0.35
54.6	C	2	A	17 48	8 18		−1.40
56.7	M	2	C	23 32	12 38		−0.55
56.7	M	2	C	23 15	11 23		−0.65
56.8	M	2	B	23 17	8 33		−0.20
57.0	M	3	A	0 39	0 16		+0.50

Key:

1. Time
2. Location
3. Group

equinox corrections E were also neglected, so that, in addition to the six improvements of the orbit elements, two additional unknowns remained, namely the equator corrections for the azimuth circle observations in Munich and Canberra, which can of course be different and were different. Eight unknowns must therefore be determined, which were determined, on the one hand, by the method of approximation described in this paper and, in addition, by the application of the formal system (2) from the normal loci of Table 1. As the mean value of $\alpha - \alpha_{\odot}$, ϱ, ξ and η the following resulted for the three groups

Group	$\alpha - \alpha_{\odot}$	ϱ	ξ	η
A	7^h36^m	0.88	−0.46	+1.04
B	9 5	0.69	−1.05	+1.00
C	11 5	0.55	−1.77	+0.43

The determination of a_i, b_i was carried out by way of approximations. The second equation (31) was written in the form

$$a_2 \sin\alpha + b_3 \cos\alpha - D = \Delta\delta - a_0 (t - t_0) \operatorname{tg}\varepsilon \cos\alpha - a_1 \operatorname{tg}\varepsilon \sin\alpha \cos\alpha - b_1 \operatorname{tg}\varepsilon \cos^2\alpha \tag{36}$$

and was used for the determination of the four unknowns a_2, b_3, D_M and D_C, under the hypothesis for the quantities a_0, a_1 and b_1 (in the first approximation, these quantities were set equal to zero), for the observations of each group. With the values which were thus obtained, the system

$$\left.\begin{aligned} E_0 + a_0 (t - t_0) + a_1 \sin\alpha + b_1 \cos\alpha - b_2 \operatorname{tg}\varepsilon \cos^2\alpha &= \Delta\alpha + a_2 \operatorname{tg}\varepsilon \sin\alpha \cos\alpha, \\ a_0 (t - t_0) \operatorname{tg}\varepsilon \cos\alpha + a_1 \operatorname{tg}\varepsilon \sin\alpha \cos\alpha + b_1 \operatorname{tg}\varepsilon \cos^2\alpha &= \Delta\delta + D - a_2 \sin\alpha - b_3 \cos\alpha \end{aligned}\right\} \tag{37}$$

was solved for the five unknowns E_0, a_0, a_1, b_1 and b_2, in that, on the right sides, the correction terms were calculated with the results of the previous approximation. If the material of the observations on the right ascensions were quantitatively equal to the declinations, it would be better to use only the first equation (37) for the determination of the five unknowns; however, it proved to be necessary to also utilize the second equation. The method was iterated until the values of the unknowns no longer changed.

In the determination of the two quantities D from (36), it must be observed that, while the coefficients a_2 and b_3 are different in each group, the values of D_M and D_C must however be equal. In order to obtain the best value of the two D within the scope of the error theory, the normal equations which were obtained from (36), were reduced in each group by elimination until only the two unknowns D_M and D_C remained; these equations from all three groups were added and their solution resulted in the associated values of D_M and D_C in each stage of the approximation.

The formal calculation in accordance with formulas (2) resulted in the following system of equations of condition:

	$+1.41\ \Delta M_0$	$-0.81\ \Delta\mu$	$+2.90\ \Delta\varphi$	$+1.49\ \Delta s$	$-0.36\ \Delta J$	$-0.04\ \Delta K$	$= -1''37$
	$+1.07$	-0.77	$+2.14$	$+1.16$	-0.19	-0.06	$= -0.88$
	$+1.50$	$+0.08$	$+0.28$	$+1.81$	$+0.23$	$+0.62$	$= -2.22$
	$+1.39$	-0.08	$+0.27$	$+1.69$	$+0.19$	$+0.59$	$= -2.73$
	$+1.05$	-0.34	-0.15	$+1.28$	$+0.08$	$+0.52$	$= -3.12$
	$+2.18$	$+1.62$	-4.28	$+2.44$	-0.73	$+0.70$	$= -0.88$
	$+1.53$	$+0.28$	-3.29	$+1.61$	-0.74	$+0.36$	$= +0.15$
$-D_M$	$+1.34\ \Delta M_0$	$-0.51\ \Delta\mu$	$+1.54\ \Delta\varphi$	$+0.99\ \Delta s$	$+1.37\ \Delta J$	$-2.01\ \Delta K$	$= -0''40$
$-D_M$	$+1.06$	-0.53	$+1.02$	$+0.79$	$+1.40$	-1.49	$= +0.23$
$-D_M$	$+0.48$	$+0.17$	$+0.23$	$+0.47$	$+2.22$	-0.18	$= +0.50$
$-D_M$	$+0.35$	$+0.04$	$+0.20$	$+0.35$	$+1.77$	$+0.21$	$= 0.00$
$-D_M$	$+0.19$	-0.02	$+0.15$	$+0.19$	$+1.35$	$+0.47$	$= +0.50$
$-D_M$	-0.48	$+0.06$	-0.18	-0.58	$+0.75$	$+1.87$	$= 0.00$
$-D_M$	-0.42	$+0.12$	-0.11	-0.51	$+0.54$	$+1.79$	$= +0.40$
$-D_M$	-0.33	$+0.18$	$+0.01$	-0.45	$+0.22$	$+1.56$	$= +0.95$
$-D_M$	-0.65	-0.07	$+0.76$	-0.77	-0.55	$+1.86$	$= +0.65$
$-D_M$	-0.61	-0.03	$+0.73$	-0.73	-0.61	$+1.76$	$= +0.70$
$-D_M$	-0.58	0.00	$+0.74$	-0.69	-0.69	$+1.55$	$= +0.40$
$-D_M$	-0.86	-0.61	$+1.76$	-0.94	-1.88	-1.61	$= +0.65$
$-D_M$	-0.83	-0.43	$+1.70$	-0.88	-1.88	$+1.32$	$= +0.50$
$-D_M$	-0.78	-0.33	$+1.62$	-0.83	-1.88	$+1.15$	$= -0.40$
$-D_C$	$+0.13$	$+0.16$	-0.24	$+0.12$	-3.14	$+0.67$	$= +0.30$
$-D_C$	$+0.10$	$+0.30$	$+0.08$	-0.17	-3.34	$+0.04$	$= -0.05$
$-D_C$	$-0.24\ \Delta M_0$	$-0.40\ \Delta\mu$	$+0.30\ \Delta\varphi$	$-0.21\ \Delta s$	$-3.14\ \Delta J$	$-0.67\ \Delta K$	$= +0''35$
$-D_C$	-0.19	-0.25	$+0.18$	-0.16	-2.58	-1.27	$= -1.40$
$-D_M$	$+1.84$	$+2.76$	$+0.35$	$+1.53$	-1.09	-3.14	$= -0.65$
$-D_M$	$+1.69$	$+2.08$	$+0.51$	$+1.39$	-0.51	-3.24	$= -0.55$
$-D_M$	$+0.92$	$+0.77$	$+0.77$	$+0.78$	$+0.69$	-2.02	$= -0.20$
$-D_M$	$+0.54$	$+0.38$	$+0.86$	$+0.48$	$+1.00$	-0.89	$= +0.50$

(38)

The result showed, from the approximate calculation, that the two equator corrections for the azimuth circle observations had the values $D_M = +0''02$ and $D_C = +0''31$. Of the other unknowns, b_2 was very unreliable in all cases and could not be determined at all for groups B and C. The results are

	E_0	a_0	a_1	a_2	b_1	b_2	b_3
Group A	$+0''11$	$+0''24$	$-2''42$	$+0''61$	$+1''77$	$+0''68$:	$-0''05$
B	-3.06	$+0.08$	$+0.85$	-0.01	$+0.53$	unb.	-0.45
C	-1.95	-0.06	-0.92	-0.56	-0.34	unb.	-0.55

Neglecting the corrections of the earth orbit and of E, the improvements in the orbit elements of Mars must be found, in accordance with (32) from these values of a_i, b_i. In Table 2, the results are compared with those of the solution of the completely formal system (38).

Table 2. Comparison of Approximated and Formal Results

1 Unbekannte	strenger Wert 2	genäherter Wert 3
	″ ″	″ ″
$\Delta L_{\odot}$	−0.57±0.94	−0.82±0.66
$\Delta\mu$	−0.07±0.24	+0.01±0.25
$\Delta\nu$	−0.30±0.12	−0.15±0.19
$\Delta\varphi$	−0.27±0.14	−0.05±0.19
ΔJ	+0.12±0.16	+0.03±0.14
ΔK	0.00±0.18	−0.17±0.13
D_M	−0.10±0.19	+0.02±0.14
D_C	+0.12±0.52	+0.31±0.40

Key:

1. Unknown
2. Formal value
3. Approximated value

With consideration of the mean error, the agreement is good in all cases. Even if, with three times the observation material, the mean error would be smaller by a corresponding factor, factual differences would still not occur between the two solutions.

The fact that, in the case of most unknowns, the approximation method results in considerably smaller mean errors is of course only apparent. The approximation method is based on the solution of systems with few unknowns, where, formally, the mean errors must be small. It is however known that, as a rule, these methods provide fairly correct values of the unknowns, even though their accuracy is stated too positively. In conclusion, it can be said that the neglects, on which the approximation methods is based, appear justified.

7. EVALUATION OF THE MUNICH OBSERVATIONS OF LARGE PLANETS

The observations of Jupiter and Saturn, which were made with the Munich instruments, were also evaluated in accordance with the described method. Because of the short orbital arc, which was passed through in the years 1941 to 1956, only a summary treatment was possible for Uranus and Neptune. In the subsequent work, Mr. Petri reports on the use of the observations of the small planets Ceres, Pallas, Juno and Vesta.

The observations are published in the same publications (6), (7), (8) as the Mars observations referred to above. The declinations observed on the meridian circle were not used because of the accuracy which is inferior to that of the azimuth circle observations. Corrections because of the variability of the measure of time were not applied in any case. As in the case of Mars, the $\Delta\delta$ of the declinations measured on the azimuth circle are different from the values published in the catalog, because the latter are already corrected due to the systems corrections which are only derived here.

A division of the material by the various values of $\alpha - \alpha_{\odot}$ was not made in the case of Jupiter and Saturn because of the insignificance of the influence of the corrections of the earth orbit. All observations which were made during an opposition were summarized in a normal locus. Tables 3 and 4 present the normal loci thus obtained.

For the determination of the unknowns, the approximation method described for the Mars observations was used, consisting of an iteration of teh equations (36) and (37). In the case of Jupiter, finally, the right ascension of the year 1948 had to be omitted because it could not be reconciled with the other observations in any manner. The specific observations were not made with the meridian circle, but with a small passage instrument, where large errors are possible in individual cases. t_0 = 1941.0 and 10 years as the time unit was selected in the terms with a_0. Both in the case of Jupiter and Saturn, the coefficient b_2 resulted only with considerable uncertainty. The final numerical results were

	E_0	a_0	a_1	a_2	b_1	b_2	b_3	D_M	D_C
Jupiter	$-2''.02$	$+0''.73$	$+0''.55$	$-0''.53$	$-0''.29$	$+0''.21$	$-0''.33$	$+0''.05$	$-0''.66$
Saturn	-1.78	$+2.37$	$+0.13$	$+0.29$	$+3.14$	$+0.03$	-0.25	$+0.02$	-1.17

The evaluation of these numbers will be carried out in the next section in connection with a discussion of the results of all planets.

Table 3. Normal Loci for Jupiter

Rektaszensionen (1)				Deklinationen (2)				
(3) Zeit	α	$\Delta\alpha$	n	(3) Zeit	(4) Ort	α	$\Delta\delta$	n
	h m	″				h m	″	
1943.2	7 8	−1.45	6	1941.1	M	2 24	−0.72	4
44.3	9 23	−1.25	6	43.2	M	7 9	−0.62	6
48.6	17 15	−4.07	7	44.3	M	9 22	−0.43	6
49.6	19 50	−1.52	6	46.4	M	13 8	+0.40	4
50.8	22 4	−2.75	6	47.5	M	15 4	+0.60	3
51.8	0 27	−1.10	6	49.7	M	19 39	+0.25	6
				50.7	M	22 9	−0.10	3
				51.9	M	0 20	−0.33	6
				53.0	M	2 37	+0.05	6
				55.1	C	7 43	+0.08	7

Table 4. Normal Loci for Saturn

Rektaszensionen (1)				Deklinationen (2)				
(3) Zeit	α	$\Delta\alpha$	n	(3) Zeit	(4) Ort	α	$\Delta\delta$	n
	h m	″				h m	″	
1943.1	4 17	+0.27	6	1941.1	M	2 26	+0.65	4
44.1	5 17	−0.33	8	43.1	M	4 17	+0.58	5
48.3	9 15	−2.38	6	44.2	M	5 16	+0.42	5
49.3	10 10	−2.68	9	48.3	M	9 15	+0.70	4
51.3	11 50	−2.59	6	49.3	M	10 11	+0.62	5
52.3	12 38	−1.96	7	50.3	M	11 1	+0.08	5
				52.4	M	12 36	+0.40	4
				53.4	M	13 23	+0.38	4
				54.4	C	14 8	+0.77	6
				55.4	C	14 57	+1.17	6

Key:

1. Right ascensions
2. Declinations
3. Time
4. Location

Because of the short orbital arc, a determination of the elements for Uranus and Neptune was impossible. Only the declinations were illustrated through a power series in accordance with time

$$\left.\begin{array}{ll} \Delta\delta = a + b\,(t - t_0) + c\,(t - t_0)^2 & \text{for Munich observations} \\ \Delta\delta = a + b\,(t - t_0) + c\,(t - t_0)^2 + d & \text{for Canberra observations} \end{array}\right\} \quad (39)$$

Here again, T_0 = 1941.0 and 10 years was selected as the time unit. Following some experiments with Neptune, the square coefficient c proved to be indeterminable and was then set equal to zero. The term d, which occurs in the observations in Canberra, is equal to the difference $D_M - D_C$ of the two equator corrections. In the same manner as for Jupiter and Saturn, all observations made during an opposition were summarized to a

normal locus; these normal loci are shown in Table 5.

Table 5. Normal loci for Uranus and Neptune

Uranus				Neptun			
1 Zeit	2 Ort	$\Delta\delta$	n	1 Zeit	2 Ort	$\Delta\delta$	n
1941.1	M	+1.08"	5	1941.4	M	−2.04"	6
43.1	M	+1.44	5	42.3	M	−1.75	8
44.2	M	+1.18	5	43.4	M	−1.54	5
48.2	M	+1.60	5	44.4	M	+1.84	5
49.2	M	0.00	3	46.4	M	−2.50	2
50.2	M	+0.70	5	49.4	M	−2.70	2
52.3	M	+0.37	3	50.5	M	+2.85	4
53.2	M	+0.76	5	52.4	M	−2.83	3
55.1	C	+0.60	6	53.4	M	+2.55	4
				54.4	C	−2.28	5
				55.4	C	+3.08	6

Key:

1. Time
2. Location

These values were subjected to an adjustment in accordance with formulas (39): the following results were obtained

	Uranus	Neptun
a	$+1''.28 \pm 0''.42$	$+1''.70 \pm 0''.17$
b	-0.27 ± 2.43	$+0.98 \pm 0.25$
c	-0.33 ± 1.45	0.00 (angenommen) (assumed)
d	$+0.26 \pm 0.75$	-0.41 ± 0.32.

The values d provide additional information about equator corrections, though very unreliable, and were used in the catalog of the azimuth circle observations (6) in this manner.

8. DISCUSSION OF THE RESULTS OF ALL PLANETS AND COMPARISON WITH THE RESULTS OF THE COMPUTER

The corrections of all orbit elements and the system improvements D_M, D_C and E must be determined from the values of the unknowns, which were obtained for the individual planets. The results found by Petri in the subsequent work concerning the observations of the small planets Ceres, Pallas, Juno, and Vesta can also be used for this calculation.

The following values have resulted for the equator corrections:

Planet	* München (D_M)	Canberra (D_C)	*Munich
Mars	+0".02	+0".31	
Jupiter	+0.05	−0.66	
Saturn	+0.02	−1.17	
Ceres	−0.32	−0.91 (1954)	
		−0.37 (1955)	
Pallas **	unbest.	−0.70	** undetermined
Juno	+0.09	+0.73	
Vesta	−0.21	−0.51	

The amounts for the equator correction D_M for Munich are small and scatter little; those for Canberra are large and scatter considerably. It is shown that D_C is largely a function of the zenith distance; for this reason two separate D_C values were derived for Ceres, which was observed in Canberra as the only planet in two oppositions with significantly different declination. In the catalog of the azimuth circle observations (6), a correction was derived, a.o. from the results of the planet observations because of the higher terms of the law of curvature.

The determination of E was faced with the difficulty that the coefficient b_2 could not be determined in most cases and was only unreliable in any case. For this reason, the second equation (34) was utilized. For the four unknowns ΔJ, ΔK, $\Delta\varepsilon$ and E, the equations

$$\left.\begin{aligned} p_1\,\Delta J + q_1\,\Delta K + \xi\,\Delta\varepsilon &= a_2, \\ q_1\,\Delta J - p_1\,\Delta K - \eta\,\Delta\varepsilon &= b_2, \\ q_1\,\Delta J - p_1\,\Delta K - \eta\,\Delta\varepsilon + E\,\mathrm{tg}\,\varepsilon &= b_3 - E_0\,\mathrm{tg}\,\varepsilon \end{aligned}\right\} \quad (40)$$

are thus available, which must be established for each group, into which the material of the observations was subdivided corresponding to the value of $\alpha - \alpha_{\odot}$. If the value of b_2 is uncertain or not sufficiently reliable, the second equation is omitted for the particular group.

In the case of all planets, the resolution in accordance with all four unknowns was so significantly uncertain that the indirect approach was used. Under the first hypothesis $\Delta\varepsilon = E = 0$,

temporary corrections ΔJ and ΔK were determined for each planet and the remainders were again solved for the unknowns $\Delta\varepsilon$ and E. Because of too great an uncertainty of the coefficients, this calculation was not performed for Juno. For Jupiter and Saturn, E was calculated from the formula

$$E \operatorname{tg} \varepsilon = b_n - E_0 \operatorname{tg} \varepsilon - b_2$$

which is evident in accordance with (40). Values of $\Delta\varepsilon$ cannot result in the case of Jupiter and Saturn because the corrections of the earth orbit were here neglected from the beginning. The following results were obtained:

Table 6. Results for $\Delta\varepsilon$ and E

Planet:	Jupiter	Saturn	Mars	Ceres	Pallas	Vesta	Mittel Mean
$\Delta\varepsilon$	—	—	$+0\overset{''}{.}03$	$-0\overset{''}{.}19$	$+0\overset{''}{.}44$	$+0.17$	$+0\overset{''}{.}11 \pm 0\overset{''}{.}13$
E	$+0\overset{''}{.}68$	$+1\overset{''}{.}13$	-0.02	-0.12	$+0.21$	$+0.75$	$+0.34 \pm 0.21$

Because of the lesser accuracy of the observations, Jupiter and Saturn were assigned half the weighting in the determination of the mean values last shown in Table 6. The two mean errors are undoubtedly too optimistic because, in the method of approximation which is used, the values of the unknowns become quite accurate, but the mean errors become too small. With the values of $\Delta\varepsilon$ and E, which were obtained, equations (40) were again solved for the two unknowns ΔJ and ΔK for each planet.

The indirect method also had to be applied for the determination of the other orbit elements in accordance with formulas (32) by first deriving preliminary values of the planet elements for each planet, while ignoring the correction quantities of the earth orbit and then subsequently determining the earth orbit quantities from the remainders. The values

$$\Delta h' = +0\overset{''}{.}07 \pm 0\overset{''}{.}08\,,$$
$$\Delta k' = +0.01 \pm 0.05\,,$$

resulted, of which the mean errors simulate a degree of accuracy which is too optimistic for reasons which have already been discussed.

In the determination of $\Delta L'$, the prerequisite of the appromation method is not fulfilled, according to which the unknown $\Delta L'$, which is neglected in the first approximation, must be small with respect to the others. For this reson, the first two equations (32) were set up for each planet and ΔL_0 and $\Delta\mu$ were eliminated from the normal equations; the following four final equations resulted for $\Delta L'$

Mars	$+0.17\,\Delta L'$	$= +0''.33,$
Ceres	$+0.03$	$= +0.10,$
Pallas	$+0.01$	$= +0.07,$
Vesta	$+0.05$	$= +0.03.$

Through the addition of these four equations, the result was $\Delta L' = +2.04 \pm 1''68$. The corrections of the earth orbit, which were found, were inserted in the original equations and, from them, in a second approximation, the final corrections of the orbit elements of the planets were determined.

The definite results are cited in the subsequent work of Petri in Table 10, which also gives the mean errors. The same table provides the results of a fitting of the entire material of the observations by all unknowns in one fell swoop, obtained with the PERM computer of the University Munich; these results were made available to us. In general, the comparison shows good agreement. Only in three cases (ΔL_0 for Pallas, $\Delta\mu$ and $\Delta\varphi$ for Saturn), the deviation is greater than the mean error.

However, the mean errors determined by the computer are so great that the values of the unknowns cannot be guaranteed in almost all cases. It could be expected from the beginning that material consisting of somewhat more than 500 individual observations cannot do justice to the complete determination of all unknowns. The large mean errors are significantly a consequence of this excess demand on the material of the observations.

Yet, the calculation carried out with the PERM computer has the considerable value of having proved the principal solubility of as extensive a task as the simultaneous determination of 50 unknowns. In contrast, the approximation method described in this paper proves the possibility that, even without the use of a computer, which is often not available or can be used only with the use of relatively significant resources, the problem can be solved manually, when the requirements for accuracy are moderate.

The uncertainty, which is inherent in the calculation, in the case of the equinox correction E, becomes particularly clear. Because, on the one hand, only the quantity E $\tan\varepsilon$ can be determined with complete accuracy and, on the other hand, in the material of the observations, the right ascensions are small in number and have an unfortunate distribution, the significant uncertainty of E could be expected. The discrepancy between the formal value $+1''30 \pm 1''36$, found with the PERM and the value $+0''34 \pm 0''21$ is so great that neither one of them can lay claim to objective accuracy. Nevertheless, the author believes that the second value $E = +0''34$ is reliable; it has good agreement with other determinations of E and it can claim the fact of experience that the method of approximation which is used mostly results in reasonably correct values for the unknowns, but has mean errors which are too small.

The fact that the equator corrections D_M and D_C which were found have resulted in a good agreement of the declination systems of the Munich azimuth circle for the observations in Munich and Canberra is a very powerful argument for the usability of the described approximation method. Thus, it is shown that the system (31) is suitable for a simple evaluation of meridian observations of the outer planets, which is sufficiently accurate with a limited amount of material.

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